Math 116 **ACTIVITY 7:** Using the probability rules; Conditional Probability, Joint Probability

Why

Understanding the probability rules is important for both understanding the language necessary for stating statistical results and understanding the way samples are related to populations - the basis of statistical inference.
Conditional probability deals with the situation in which the possible outcomes of a random process are somehow restricted (we have found out part -but not all - of the result, or we sample from only part of the population, conditions have changed, etc.) and we want the probabilities for this modified process (This corresponds to controlling a variable in an experiment and then observing another variable, or to the predictor and response in a regression situation). For experimental/theoretical work, this is particularly important for deciding whether two events are independent or are related (can't be both). The basic *multiplication rule* for joint probabilities (used with "and" situations) is based on conditional probability—with a special case for independent events.

LEARNING OBJECTIVES

1. Learn to use the basic probability rules in combinations in different settings
2. Be able to use a tree diagram for a multi-step probability question
3. Work as a team, using the team roles.

CITERIA

1. Success in working as a team and in fulfilling the team roles.
2. Success in involving all members of the team in the conversation.
3. Success in completing the exercises

RESOURCES

1. The Statistics Handbook chapter on Probability—especially pp.9–12
2. Your calculators
3. The team role desk markers (handed out in class for use during the semester)
4. 40 minutes

PLAN

1. Select roles, if you have not already done so, and decide how you will carry out steps 2 and 3 (5 minutes)
2. Work through the group exercises given here - be sure everyone understands all results & procedures(25 minutes)
3. Assess the team's work and roles performances and prepare the Reflector's and Recorder's reports including team grade (5 minutes).

DISCUSSION

Standard probability formula for conditional probability: $P(A|B) = \dfrac{P(A \text{ and } B)}{P(B)}$

Here B represents the condition we know is satisfied (maybe we are choosing from this group, maybe we found out something, but not everything, about the result) and A represents the condition we are still looking for.
For example, If 30% of students are tired, 40% are worried, and 20% are both tired and worried, then if we pick a student, the probability of getting a worried student is $P(\text{worried}) = .4$, but if we pick a tired student [so we already know we have "tired"], the probability of getting a worried student is $P(\text{worried} \mid \text{tired}) = \dfrac{.20}{.30}$—about .67. That is, tired students are *more likely to be worried* [than students in general] (there is some connection—at least statistically—between being tired and being worried).
Dont confuse this with a *joint* probability situation, where we *do not know* (no control) that either condition will occur, but are looking for both conditions: The probability of getting a student who is tired and worried is a joint probability [we dont know that either condition holds were looking for both] $P(\text{tired and worried}) = .20$

(as given in the information above - notice it is *smaller* than both P(tired) and P(worried)).

General multiplication rule for probabilities: $P(A \text{ and } B) = P(A|B) * P(B) = P(B|A) * P(A)$

For example, if 20 % of all fleas are large, and 40% of all large fleas are cute, then the proportion of (all) fleas that are large, cute fleas is $.2 * .4 = .08 = 8\%$. If we also know that 20% of all not-large fleas are cute, then we know that the proportion of (all) fleas that are not-large but cute is $.8 * .2 = .16$, and so the proportion of all fleas that are cute is $.08 + .16 = .24$. [We need to put the large & cute together with the not-large and cute to get all of the cute]

This formula is a rewrite of the formula for conditional probability—the two formulas give the relation between *joint* ("and") probabilities and *conditional* ("if") probabilities. (We also get a relation between the two conditional probabilities $P(A|B)$ and $P(B|A)$—they are not at all the same).

Independence (and relationship):

Two events A and B are *independent* if $P(A|B) = P(A)$ [conditional probability for A = unconditional probability for A]—in which case it will also be true that $P(B|A) = P(B)$. Otherwise they are *related* (either positively or negatively—but we may not know why).

EXERCISE

1. At the College of Knowledge, 70% of the students receive financial aid, 60% of the students are commuters, and 80% of the commuters receive financial aid [Notice that this 80% is a conditional probability].
   (a) If we pick a student at random, what is the probability we will get a student who does not receive financial aid?
   (b) If we pick a student at random, what is the probability we will get a student who is a commuter and receives financial aid [This is not a conditional probability] ?
   (c) If we pick a student who receives financial aid at random, what is the probability we will get a commuter [You need the answer from b.), for this]?
   (d) If we pick a student at random, what is the probability we will get a student who is a commuter or receives financial aid?

2. Here is a table with information on the 40 students enrolled in another course at the College of Knowledge

		Major				
		English	Theater	Philosophy	Other	Totals
Year	Sophomore	6	4	5	2	17
	Junior	5	4	0	7	16
	Senior	1	2	2	2	7
	Totals	12	10	7	11	40

   (a) If we select one of these students at random, what is the probability of getting a Junior?
   (b) If we select a Philosophy major at random, what is the probability of getting a Junior?
   (c) Are the events "Get a Junior" and "Get a Philosophy Major" independent? Explain.
   (d) Are the events "Get a Junior" and "Get a Theater major" independent? Explain.
   (e) If two students (two distinct students) are chosen, what is the probability of getting exactly one Senior [either the first or the second is a senior—but not both]?

READING ASSIGNMENT (in preparation for next class)
Read the section on tree diagrams

SKILL EXERCISES:(hand in - individually - with assignment for this week)
Exercises for Chapter 2, #5, 7–10