Math 116 **ACTIVITY 5:** Separable Differential Equations

Why

We now look at our final class of differential equations that can be solved with tools that we have developed in this course. Many growth models fake on one of the forms we consider, extending the importance of the relation between rate of change and value of any quantity.

LEARNING OBJECTIVES

1. Be able to recognize a separable differential equation.
2. Be able to solve and interpret the solution of a separable differential equation or initial-value problem with a separable equation.
3. Be able to write the differential equation (of one of our nice forms) for a situation involving rates of change.

CITERIA

1. Success in working as a team and in fulfilling the team roles.
2. Success in involving all members of the team in the conversation.
3. Success in completing the exercises

RESOURCES

1. Your text and class notes (especially the examples)
2. The team role desk markers (handed out in class for use during the semester)
3. 40 minutes

PLAN

1. Select roles, if you have not already done so, and decide how you will carry out steps 2 and 3 (5 minutes)
2. Work through the group exercises given here - be sure everyone understands all results & procedures(25 minutes)
3. Assess the team's work and roles performances and prepare the Reflector's and Recorder's reports including team grade (5 minutes).

DISCUSSION

For an "elementary" differential equation, the "y" does not appear in the equation and we can write the equation maybe using some algebraic manipulation) in the form $y' = f(x)$, so we can get a general solution (by taking antiderivative on both sides of the equation) $y = \int f(x)dx$ and a particular solution of an initial-value problem by solving for (and then substituting) a value of C, the constant of integration.

For a "linear" differential equation, we can rewrite the equation (algebraically) in the form $y' + p(t)y = q(t)$ an obtain a general solution using an integrating factor $G(t) = e^{\int p(t)dt}$ to convert the left side to the derivative of $G(t)y$ and then take antiderivative on both sides to obtain $G(t)y = \int q(t)G(t)dt$, so that $y = \dfrac{1}{G(t)} \displaystyle\int q(t)G(t)dt$

For the "autonomous" differential equations, where the independent variable t (or x) does not appear, we can consider equilibrium solutions and stabiilty, but we have not yet discussed solution methods. The methods used are those used for "separable" equations, which we now consider.

A *separable* differential equation is one which can be rewritten algebraically into the form (y-stuff)y' = (t-stuff); formally, as $f(y)y' = g(t)$—the two variables x and y are "separated" onto the two sides of the equation, with all the y terms *multiplied by* the derivative y'.
Examples: $yy' = x + 3$ is separable (already separated).

$y' = xy$ is separable (can be written $\frac{1}{y}y' = x$).

$y' = .2y$ is separable (can be written $\frac{1}{y}y' = .2$. The t-stuff is the constant .2).

$y' = \cos(y + t)$ is not separable

$y' + y = t$ is not separable (We need $f(y)$ TIMES y'. Dividing by y will give a $\frac{t}{y}$ so t and y aren't separated).

Notice that every y-expression must be *multipllied by* the y'–our solution method depends on that.

To *solve* a separable differential equation, we first write it in the standard form $f(y)y' = g(t)$ and then take antiderivative of both sides with t as variable (if our variable is x, we'll use that, of course). It's this antiderivative step that requires every y term to be multiplied by y'—our substitution rules (based on the chain rule) require it.

Thus: For $yy' = x+3$ we get $\int yy'dx = \int x+3dx$, so $\frac{y^2}{2} = \frac{(x+3)^2}{2} + C$ so $y = \sqrt{(x+3)^2+2C}$ or $y = -\sqrt{x+3)^2+2C}$ (Need initial condition to know if sign should be + or −, as well as to find C). For the initial-value problem $yy' = x+3, y(0) = -2$, we would know we must take the negative square root (positive square root couldn't be −2) and we would have $-2 = -\sqrt{(0+3)^2+2C}$, so $3^2+2C = 4$ and so $2C = -5$. Our solution is $y = -\sqrt{(x+3)^2-5}$.

For $y' = .2y$, $y(0) = 30$, we get $\frac{1}{y}y' = .2$ (as long as y is never 0—we'll deal with that situation tomorrow) so $\int \frac{1}{y}y'dt = \int .2dt$, which gives $\ln|y| = .2t + C$ Applying the exponential function on both sides gives $|y| = e^{.2t+C} = e^{.2t}e^C$. Since $y(0) = 30$ (positive) we know that y will be positive, so $|y| = y$ and we have $30 = e^Ce^{.2(0)} = e^C$ So $e^C = 30$ (we don't really care about C—it's e^C that's important for our solution) so our solution is $y = 5e^{.2t}$. This is where we got the exponential function last semester.

EXERCISE

1. For each of these initial value problems, indicate whether the equation is an elementary differential equation, a linear differential equation, a separable differential equation or none of these (and thus not solvable by our methods). Do not solve.
   (a) $y' = 2xy + x^2, y(0) = 10$
   (b) $y' = 4xy^2, y(0) = 20$
   (c) $y' + t^2 = \cos t, y(\pi) = 8$
2. Solve - show the steps - and give the function value at the point specified. Do *not* worry about constant solutions, for today.
   (a) $y' = \frac{x}{y}, y(0) = 3$ Give solution and give $y(4)$
   (b) $y' = 2x(y+1), y(0) = 5$ Give solution and give $y(1)$
3. For the autonomous differential equation $y' = y^2 - 4$, give the equilibrium solutions and indicate the stability of each.

READING ASSIGNMENT (in preparation for next class)
Re-read 8.4, noticing the information on *constant* solutions and on *implicit* solutions.

SKILL EXERCISES:(hand in - individually - with assignment for this week)
p. 581 # 1, 2, 3, 19, 20 (to separate—factor out an x on the right-hand side, divide by the factor with the y), 36