Math 116   ACTIVITY 4: Differential equations and change models

Why

Calculus is about change and rates of change. Differential equations relate rate of change of a quantity (population size, concentration of a solution, temperature) to the value of the quantity and/or to time, and express many of the basic relationships of growth. Understanding differential equations involves both the mechanics (solving, checking solutions) and applications (representing relationships, interpreting results).

LEARNING OBJECTIVES

1. Be able to recognize a solution of a differential equation.

2. Be able to solve and interpret the solution of a differential equation of one of our "nice" types: of the form $y' = f(t)$ or of the form "linear, of order 1"

3. Be able to write the differential equation (of one of our nice forms) for a situation involving rates of change–particularly the single-compartment model.

CITERIA

1. Success in working as a team and in fulfilling the team roles.

2. Success in involving all members of the team in the conversation.

3. Success in completing the exercises

RESOURCES

1. Your text and class notes (especially the examples)

2. The team role desk markers (handed out in class for use during the semester)

3. 40 minutes

PLAN

1. Select roles, if you have not already done so, and decide how you will carry out steps 2 and 3 (5 minutes)

2. Work through the group exercises given here - be sure everyone understands all results & procedures(25 minutes)

3. Assess the team's work and roles performances and prepare the Reflector's and Recorder's reports including team grade (5 minutes).

EXERCISE

1. In each of these a differential equation and several functions are given. Determine which functions (may be more than one) are solutions of the given equation. Note that you are *not* asked to *solve* the equation but to check each of the functions given.

   (a) Equation $y' = 2xy + y$ Functions: 1.) $y = 8e^{x^2+x}$ 2.) $y = 5e^{x^2}$ 3.) $y = 5e^{x^2+x+4}$

   (b) Equation $y' = 4xy^2$ Functions 1.) $y = \dfrac{-1}{2x^2+5}$ 2.) $y = \dfrac{5}{2x^2}$ 3.) $y = -2x^{-2}$

2. Solve - show the steps - and give the function value at the point specified.

   (a) $2y' - 6y - e^x = 0, y(0) = 5$ Give solution and give $y(\ln(3))$
   (b) $xy' - 4x^4 = -3x^3 + x^2, y(1) = 0$ Give solution and give $y(4)$

3. Write the differential equation for the amount of salt present at each time t minutes in this situation and solve to give the amount of salt at each time t min.
   Also answer: When is the tank filled? How much salt is present when the tank becomes full?
   A 2000 gallon tank starts with 200 lb. of salt dissolved in 500 gal. of water. Water containing 1 lb of salt per gallon is flowing into the tank at 4 gal/min. and the well-mixed solution is flowing out at 2 gal/min.
   NOTE: it will be helpful in writing the work to remember/realize that $e^{a\ln(u)} = e^{(\ln(u))(a)} = u^a$

READING ASSIGNMENT (in preparation for next class)
In your text, read section 8.3: Autonomous differential equations and stability

SKILL EXERCISES:(hand in - individually - with assignment for this week)
p. 560 # 15, 16, 31, 32, 40, 42, 47, 51