Math 116 **ACTIVITY 2:** Antiderivative (indefinite integral) techniques: Substitution and Integration by parts

Why

Calculation of antiderivatives is useful both for its own sake (especially in solving differential equations—coming soon) and for calculation of (definite) integrals. The basic techniques grow directly out of the basic derivative rules. The "direct" rules (for powers, exponentials, etc.) are very limited in their application, so we need to add the effects of the chain rule—which gives us our "substitution" technique—and the product rule–which gives us integration by parts—to deal with most of the basic antiderivative situations.

LEARNING OBJECTIVES

1. Be able to recognize the situations that allow use of the basic formulas with the substitution technique.
2. Be able to recognize when substitution is not enough but integration by parts will work
3. Know how to use these two techniques–identifying the "parts" (u, u', v', du, etc.), matching the result to the problem, writing and using the results

CITERIA

1. Success in working as a team and in fulfilling the team roles.
2. Success in involving all members of the team in the conversation.
3. Success in completing the exercises

RESOURCES

1. The course syllabus
2. The team role desk markers (handed out in class for use during the semester)
3. 40 minutes

PLAN

1. Select roles, if you have not already done so, and decide how you will carry out steps 2 and 3 (5 minutes)
2. Work through the group exercises given here - be sure everyone understands all results & procedures(25 minutes)
3. Assess the team's work and roles performances and prepare the Reflector's and Recorder's reports including team grade (5 minutes).

DISCUSSION

We have modified our basic antiderivative rules in the light of the chain rule. Here we have the basic rules, with the notational convention that du represents $u'dx$ or $u'dt$ (depending on the variable).:
A. The basic antiderivative rules

1. Constant $\int k\ du = ku + C$ for any constant k
2. Power (nice case): $\displaystyle\int u^n\ dx = \frac{u^{n+1}}{n+1} + C$ when $n \neq -1$
3. Power (less-nice case): $\displaystyle\int u^{-1}\ dx = \int \frac{1}{u}\ dx = \ln|u| + C$
4. Exponential: $\displaystyle\int e^u\ du = e^u + C$
5. Sine: $\displaystyle\int \sin u\ du = -\cos u + C$
6. Cosine: $\displaystyle\int \cos u\ du = \sin u + C$ for any constant a

B: Reduction rules

1. Constant coefficient: $\int kf(x)dx = k\int f(x)dx$ for any constant k
2. Sum/difference: $\int f(x) \pm g(x)dx = \int f(x)dx \pm \int g(x)dx$

C. We have now added the *integration by parts* rule (to be tried if attempts to use substitution leave "extra parts" unexplained.)
$\int u\ dv = uv - \int v\ du$

To calculate an antiderivative, we need to identify the formula/rule to be used, identify the parts (u, v, u', etc. as appropriate to the formula) and use this to write the result. Remember that substitution should be attempted before parts, and that a constant (but not variables) in the coefficient can be adjusted.

Some examples:

1. $\displaystyle\int \frac{5x^2}{(4x^3-4)^3}dx = 5\int x^2\frac{1}{(4x^3-4)^3}dx = 5\left(\frac{1}{12}\right)\int 12x^2(4x^3-4)^{-3}dx = \frac{5}{12}\frac{(4x^3-4)^{-2}}{-2} + C$
$\displaystyle= -\frac{5}{24(4x^3-4)^2} + C$
Uses $\displaystyle\int u^n du = \frac{u^{n+1}}{n+1} + C$ with $u = (4x^3-4), n = -3$ (so $u' = 12x^2$—the coefficient had to be adjusted)

2. $\displaystyle\int \frac{\ln(2x+2)}{x+1}dx = \int \ln(2x+2)\frac{1}{x+1}dx = \int \ln(2x+2)\frac{1}{x+1}dx = \frac{(\ln(2x+2))^2}{2} + C = \frac{(\ln(2x+2))^2}{2} + C$
Uses $\displaystyle\int u^n du = \frac{u^{n+1}}{n+1} + C$ with $u = \ln(2x+2), n = 1$ so $u' = \frac{1}{2x+2}(2) = \frac{1}{x+1}$—the coefficient did not need adjustment.

3. $\displaystyle\int x\sqrt{5x+3}dx = \int x(5x+3)^{\frac{1}{2}}dx = x\left(\frac{2}{15}(5x+3)^{\frac{3}{2}}\right) - \int \frac{2}{15}(5x+3)^{\frac{3}{2}}dx$
$\displaystyle= \frac{2x(5x+3)^{\frac{3}{2}}}{15} - \frac{4(5x+3)^{\frac{5}{2}}}{375} + C$
First step: Can't use $\displaystyle\int u^n du$ because u would be $5x+3$, so u' would be 5 — we have an extra x there.
Try parts $\displaystyle\int udv = uv - \int vdu$ with
$u = x, v' = (5x+3)^{\frac{1}{2}}$
$\displaystyle u' = 1, v = \int v'dx = \int (5x+3)^{\frac{1}{2}}dx = \frac{1}{5}\int 5(5x+3)^{\frac{1}{2}}dx = \frac{1}{5}\frac{(5x+3)^{\frac{3}{2}}}{\frac{3}{2}} = \frac{2}{15}(5x+3)^{\frac{3}{2}}$ which gives second step.
For final integral, $\displaystyle\int \frac{2}{15}(5x+3)^{\frac{3}{2}}dx = \frac{2}{15}\left(\frac{1}{5}\right)\int 5(5x+3)^{\frac{3}{2}}dx = \left(\frac{2}{15}\right)\left(\frac{1}{5}\right)\frac{(5x+3)^{\frac{5}{2}}}{\frac{5}{2}} + C$

4. $\displaystyle\int x^2\cos(x)dx = x^2\sin(x) - \int (2x)\sin(x)dx = x^2\sin(x) - 2\int x\sin(x)dx$
$\displaystyle= x^2\sin(x) - 2\left(x(-\cos(x)) - \int -\cos(x)dx\right) = x^2\sin(x) + 2x\cos(x) - 2\sin(x) + C$
Uses integration by parts twice. First integral has an "extra" x^2 so we can't use $\displaystyle\int \cos udu$. We try parts with:
$u = x^2, v' = \cos(x)$
$u' = 2x, v = \int\cos(x)dx = \sin(x)$. For the integral $\displaystyle\int \sin(x)dx$ we use:
$u = x, v' = \sin(x)$
$u' = 1, v = \displaystyle\int \sin(x)dx = -\cos(x)$ and for the final integral we have
$\displaystyle\int -\cos(x)dx = -\sin(x) + C$

EXERCISE Calculate these antiderivatives. Some will involve integration by parts (and possible two steps). You should expect all to involve use of substitution. For each, check your result by taking the derivative.

1. $\displaystyle\int e^{3x}\sin(e^{3x})\ dx$

2. $\displaystyle\int x(\ln(x))^2\ dx$

3. $\displaystyle\int \frac{\cos(5t)}{\sin(5t)+9}\,dt$

4. $\displaystyle\int x^2\sqrt{2x^3+5}\,dx$

5. $\displaystyle\int (2x+1)e^{3x^2+3x}\,dx$

6. $\displaystyle\int x^2e^{-x}\,dx$

READING ASSIGNMENT (in preparation for next class)
In your text, read section 5.7: Integration techniques: Tables and technology (actually approximations)

SKILL EXERCISES:(hand in - individually - with assignment for this week)
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