Math 116 - 2013

Derivative and Antiderivative review

This review covers (most of) the first-semester material on limits, derivatives, and antiderivatives that we will call upon for second semester (doesn't include much on graphs, on log & exponential, exponential growth functions, or trig functions - which also will show up quite a bit). This list is intended to remind you of derivative ideas covered so that you can see where you need to find more practice problems - it does not include enough exercises for relearning techniques if you have trouble, but it does show the sections where you could find additional practice exercises.

Review Exercises

(should be done by 1/22/2013, but will not be collected): In Bittinger, Brand, Quintanilla:

p. 80	15, 24,
p. 91	3, 7, 18, 31, 43
p. 98	13, 25, 29
p. 112	9, 13, (note that these involve using the *definition* - not the shortcuts)
p. 123	3, 13, 23, 35, 41
p. 130	9, 13
p. 139	9, 11, 21, 95
p. 147	3, 27, 29, 33, 61, 75
p. 153	11, 39
p. 193	17, 33
p. 225	25, 43
p. 253	3, 7, 12
p. 275	15, 23, 51, 77
p. 292	41, 47, 55
p.308	9,
p. 345	13, 15, 23, 41, 45, 63

NOTES
Limits

The limit of a function $f(x)$ as the value of x approaches a number a [written $\lim_{x \to a} f(x)$] is the number L that $f(x)$ gets close to as values of x are chosen closer and closer to a. [If there is such a number — it's possible there isn't] We have two basic approaches to finding limits:

a.) The table" approach: Make a table with two rows: values for x on the top —- arranged getting closer & closer to a (from both larger & smaller values), corresponding values of $f(x)$ on the bottom (calculated from the rule for f) — if the values on the bottom approach a single value, that is the limit.

b.) The algebraic approach: If the function is given by a *formula* near a (same formula for values a little larger as for values a little smaller), see if the formula makes sense [gives a number] when $x = a$. If it does, the number given is the limit. If not, sometimes the formula can be simplified for all values except a and this simplified formula will give a value for $x = a$ — the "almost everywhere" rule says this value *is* the limit.

Derivatives

Derivative is a rate of change (In particular, if $N(t)$ is size at time t , then $\frac{dN}{dt}$ gives growth rate) and because slope also represents rate of change, $\frac{df}{dx}$ gives the slope of the tangent to the graph of $y = f(x)$ at each point. The derivative is defined by the limit of a difference quotient, but for functions given by "elementary functions" (algebra, exponential/logarithmic functions, trig functions and their combinations) we have some shortcut rules. The "approximate and then take a limit" approach used in the definition of the derivative will reappear this semester in the definition of the integral.

A. Derivative shortcut rules (in these, u, v represent any functions of the variable x or any expressions involving the variable x

1. The big five (in general form) - the basic rules that we will see most often.

   (a) Power rule $\frac{du^n}{dx} = u^{n-1}\frac{du}{dx}$

(b) Exponential rule $\frac{de^u}{dx} = e^u\frac{du}{dx}$ (for other bases: $\frac{da^u}{dx} = a^u\frac{du}{dx}\ln a$)

(c) Logarithm rule $\frac{d\ln u}{dx} = \frac{1}{u}\frac{du}{dx}$ (for other bases: $\frac{d\log_a u}{dx} = \frac{1}{u}\frac{du}{dx}\log_a e$)

(d) Sine rule $\frac{d\sin u}{dx} = \cos u\frac{du}{dx}$

(e) Cosine rule $\frac{d\cos u}{dx} = -\sin u\frac{du}{dx}$

2. Reduction rules

(a) Constant coefficient rule: $\frac{dku}{dx} = k\frac{du}{dx}$ for any *constant* k

(b) Sum/difference rule $\frac{d(u \pm v)}{dx} = \frac{du}{dx} \pm \frac{dv}{dx}$

(c) Product rule $\frac{d(uv)}{dx} = \frac{du}{dx}v + u\frac{dv}{dx}$

(d) Quotient rule $\frac{d\frac{u}{v}}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

(e) Chain rule $\frac{df(u)}{dx} = f'(u)\frac{du}{dx} = \frac{df}{du}\frac{du}{dx}$

B. Implicit differentiation If $A = B$, then $\frac{d}{dt}A = \frac{d}{dt}B$ at all times. (Used with the chain rule to get derivatives, when we can't solve an equation for y—or [usually used with $\frac{d}{dt}$, in this case] for getting rate-of-change relations from relations on quantities) Example: If $3x^2 + \ln y - xy^2 = 8$, we can get $\frac{dy}{dx}$ at any point (x, y) on the graph, by implicit differentiation:

$\frac{d}{dx}(3x^2 + lny - xy^2) = \frac{d}{dx}8$

$3(2x) + \frac{1}{y}\frac{dyt}{dx} - (y^2 + x(2y\frac{dy}{dx}) = 0$

$6x - y^2 + \frac{1}{y}\frac{dy}{dx} - 2xy\frac{dy}{dx} = 0$

$(\frac{1}{y} - 2xy)\frac{dy}{dx} = -6x + y^2$

$\frac{dy}{dx} = \frac{y^2 - 6x}{\frac{1}{y} - 2xy} = \frac{y^3 - 6xy}{1 - 2xy^2}$ [Need to know both x and y to get numerical value — important ideas are: 1. taking derivative of both sides of equation; 2. keeping $\frac{dy}{dx}$ as a symbol (a new variable) since we don't have a formula for it during the work]

Antiderivatives

A function $F(x)$ is an *antiderivative* of $f(x)$ if $\frac{dF(x)}{dx} = f(x)$. The *general antiderivative* of a function always contains an undetermined constant (represented as C) so that it represents all antiderivatives at once. To get a *particular* antiderivative, you need to know the value of the function (antiderivative) at one point.
The antiderivative rules all come from the derivative rules, but using them is not as straightforward because the chain rule kicks in from the beginning. We use $\int f(x)dx$ to represent "the general antiderivative of $f(x)$ with x as variable." Since "the general antiderivative" of a derivative is the original function plus or minus any constant, we have $\int f'(x)dx = f(x) + C$. . The rules are often written in a shorter form in which u represents any function of x , and du represents $u'dx$ (that is, $\frac{du}{dx}dx$) thus $\int du = u + C$

A. The basic antiderivative rules

1. Constant rule $\int kdx = kx + C$ for any constant k

2. Power rule—nice case: $\int x^n dx = \frac{x^{n+1}}{n+1} + C$ when $n \neq -1$

3. Power rule—less-nice case $\int x^{-1}dx = \int \frac{1}{x}dx = \ln|x| + C$

4. Exponential rule $\int e^{ax} dx = \frac{1}{a} e^{i} ax + C$ for any constant a

5. Sine $\int \sin ax dx = -\frac{1}{a} \cos ax + C$

6. Cosine $\int \cos ax dx = \frac{1}{a} \sin ax + C$

B: Reduction rules

1. Constant coefficient rule $\int k f(x) dx = k \int f(x) dx$ for any constant k

2. Sum/difference rule $\int f(x) \pm g(x) dx = \int f(x) dx \pm \int g(x) dx$